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ON THE DETERMINATION OF PRESSURE AND DENSITY PROFILES FROM TEMPERATURE PROFILES IN THE ATMOSPHERE

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SUMMARY

The rocket grenade technique for measuring meteorological parameters in the upper atmosphere produces direct observations of temperature and wind only. Pressure and density profiles must be computed from well-known physical considerations by one of several analytical methods. Similar considerations apply to two other techniques, the falling sphere and the pitot-static tube experiments, in which density is the measured parameter and temperature and pressure must be derived. Basically, all of the analytical methods employ the hydrostatic equation and perfect gas law to relate temperature or density to pressure. Obviously, it is desirable to employ the one method which produces the most accurate results. Each method is presented and the effect of errors in the initial pressure or temperature assumption is discussed. A comparison of pitot-static tube data and rocket-grenade data is made for experiments launched almost simultaneously.

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INTRODUCTION

In the grenade experiment the barometric equation, which is a variation of the hydrostatic equation, is integrated with a known temperature profile starting with an assumed or measured initial pressure. In the falling-sphere and pitot-static tube experiments, an initial temperature or pressure point must be assumed or measured independently to permit computation of the temperature or pressure profiles, respectively, from the measured density profile. The effects of errors in these initial values are discussed for the methods used in each of these three experiments. In the falling-sphere and pitot-static tube experiments the derivation of temperature or pressure profiles is best accomplished by assuming arbitrary initial values at the maximum height of the sounding and integrating the barometric equation downward. Errors introduced by the initial values will have become negligible about 15 km below the maximum altitude. This method, however, does not apply to the derivation of density and pressure from the grenade experiment. Here the optimum method is to perform an independent initial pressure (or density) measurement at the bottom altitude of the temperature sounding and integrate the barometric equation upward. The initial measurement can be performed very accurately and any small error remains small over the entire calculated profile.

The theory and operational details of the grenade, falling-sphere, and pitot-static tube experiments have been described by experimenters such as Nordberg and Smith, Peterson and McWatters, and Horvath, Simmons, and Brace (References 1, 2 and 3), respectively. This discussion will be concerned with the methods used to derive the entire static structure of the atmosphere from measured parameters.

PITOT-STATIC TUBE EXPERIMENT

The hydrostatic equation in the standard notation is given by

$$dp = -\rho g \, dz , \tag{1}$$

where p is the atmospheric pressure at altitude z, ρ is the atmospheric density, and g is the acceleration due to gravity. Horvath, Simmons, and Brace (Reference 3) have utilized this expression in the form,

$$p_i = p_0 + \int_{z_0}^{z_i} g\rho \, dz \ , \tag{2}$$

where the subscripts 0 and i denote an initial altitude and some other altitude, respectively, which are boundaries of the layer under consideration, to compute atmospheric pressure profiles in the pitot-static tube experiment. While it is used only as a check on the calibration of the static pressure gauge, it represents a means of determining pressure from density observations alone. An error in the initial pressure, p_0, will rapidly become insignificant compared to the contribution of the integral term in Equation 2 where the integration is performed downward. However, this method makes the computed pressure profile uncertain for the top 15 km of the data.

FALLING-SPHERE EXPERIMENT

In the falling-sphere experiment the following equations were used by Jones, Peterson, Schaefer, and Shulte (Reference 4) to obtain the temperature profile from the measured density profile. The perfect gas law is given by

$$p = \rho \frac{R^*}{M} T \ , \tag{3}$$

where R^* is the universal gas constant, M is the mean molecular weight of air, and T is the absolute temperature of the air. Equation 3 is substituted into Equation 2 and divided by $\rho_i \, (R^*/M)$ to obtain the expression,

$$T_i = \frac{\int_{z_0}^{z_i} \rho g \, dz}{\rho_i \frac{R^*}{M}} + \frac{\rho_0}{\rho_i} T_0 \ . \tag{4}$$

Pressure can then be derived from the temperature and density data with the perfect-gas law. Here again, the integration is performed downward from the top of the valid density data, and an initial value of temperature, T_0, must be assumed. It is evident from Equation 4 that any error introduced by assuming an incorrect T_0 becomes negligible after the integration proceeds over an altitude of 15 km because the ratio ρ_0/ρ_i becomes small. Note that usable pressure data are obtained beginning at an altitude 15 km below the highest valid density datum.

Both the falling-sphere and the pitot-static tube experiments rely on observed density profiles to compute pressure. Clearly, in this case, use of a downward integration is indicated to minimize pressure computation errors. That the same approach does not apply to the grenade experiment will now be demonstrated.

ROCKET-GRENADE EXPERIMENT

For the grenade pressure profile computation, it has been argued that inserting an assumed standard atmosphere value of pressure at the top of the temperature profile and integrating downward would produce the most accurate pressure and density profiles. This viewpoint reasons that any error introduced in the initial reference pressure would become insignificant as the pressure increases rapidly with decreasing altitude in the fashion which has been demonstrated above.

Combining Equations 1 and 3 produces the barometric equation,

$$d \ln p = - \frac{gM}{R^*T} dz \quad . \tag{5}$$

Integrating Equation 5 from an altitude z_0 to some level below it, z_i yields

$$\ln p_i - \ln p_0 = - \frac{M\bar{g}_i}{R^*\bar{T}_i} \left(z_i - z_0\right) , \tag{6}$$

where the bar denotes the mean value in the layer. If the value of p_0 is in error by an amount δ, then Equation 6 becomes

$$\ln \left(p_0 + \delta\right) - \ln p_i = \frac{M\bar{g}_i}{R^*\bar{T}_i} \left(z_i - z_0\right) \quad . \tag{7}$$

For uniform intervals of $\left(z_i - z_0\right)$, let $M/R^* \left(z_0 - z_i\right) = K$ and let $\bar{g}_i/\bar{T}_i = \phi_i$. Then Equation 7 becomes

$$p_i = \left(p_0 + \delta\right) e^{K\phi_i} = p_0 e^{K\phi_i} + \delta e^{K\phi_i} \quad . \tag{8}$$

Equation 8 demonstrates the fact that any error in p_0 will propagate downward through the computed profile with a magnitude equal in percent to the original error in pressure. In other words, if the assumed pressure at 90 km is in error by plus ten percent, the value of computed pressure at 40 km will also be ten percent too high. As an alternate approach to compute pressure and density profiles from the temperature profiles observed with the grenade experiment, pressures or densities measured by balloon-sondes may be used as the initial values. Then integration of Equation 5 may be performed from the lowest to the highest altitude of the sounding. This method, denoted as "Method A," was applied to two grenade soundings and the results were compared to "Method B," described above where a "standard" pressure was assumed as initial value at the top of the sounding and Equation 5 was integrated from top to bottom. The actual measured temperature profiles were applied identically to both methods. Numerical results for two soundings using both methods are tabulated in Tables 1 and 2.

COMPARISON OF PITOT-STATIC TUBE DATA WITH ROCKET-GRENADE DATA

A reference had to be chosen to permit a comparison of the accuracies of Methods A and B. A standard atmosphere could have been used for reference, but since pitot-static tube soundings were made nearly simultaneously with each of the grenade soundings, the grenade experiment data were compared with the observed pitot-static tube profiles in Tables 1 and 2. The Tables show

Table 1

Wallops Island Soundings of 6 June 1962.

Height	Parameter*	Method A			Method B			Pitot-Static Data	Exp.†
		Grenade Data	Exp.†	Percent Difference From Pitot	Grenade Data	Exp.†	Percent Difference From Pitot		
36 km	pressure	.54069	3	4.78	.63509	3	23.08	.516	3
	density	.75489	-2	-0.41	.88668	-2	16.98	.758	-2
	temp.	.24953	3	5.20	.24953	3	5.20	.2372	3
40 km	pressure	.31773	3	3.50	.37320	3	21.56	.307	3
	density	.42871	-2	5.85	.50356	-2	24.34	.405	-2
	temp.	.25820	3	-2.12	.25820	3	-2.12	.2638	3
50 km	pressure	.89780	2	2.02	.10545	3	19.83	.880	2
	density	.11717	-2	5.56	.13763	-2	23.99	.111	-2
	temp.	.26693	3	-3.08	.26693	3	-3.08	.2754	3
60 km	pressure	.25304	2	5.87	.29729	2	24.39	.239	2
	density	.34720	-3	2.12	.40792	-3	19.98	.340	-3
	temp.	.25391	3	3.68	.25391	3	3.68	.2449	3
70 km	pressure	.58888	1	10.48	.69241	1	29.91	.533	1
	density	.98949	-4	11.55	.11634	-3	31.16	.887	-4
	temp.	.20734	3	-1.13	.20734	3	-1.13	.2097	3
80 km	pressure	.10668	1	5.62	.12548	1	24.24	.101	1
	density	.21169	-4	13.20	.24899	-4	33.15	.187	-4
	temp.	.17556	3	-6.27	.17556	3	-6.27	.1873	3
90 km	pressure	.15329	0	-1.74	.18025	0	15.54	.156	0
	density	.32042	-5	1.40	.37677	-5	19.23	.316	-5
	temp.	.16667	3	-3.10	.16667	3	-3.10	.1720	3
93 km	pressure	.81629	-1	-5.08	.95977	-1	11.60	.860	-1
	density	.19013	-5	8.65	.22355	-5	27.74	.175	-5
	temp.	.14957	3	-12.74	.14957	3	-12.74	.1714	3

*Pressure in newtons/m^2, density in kg/m^3, temperature in °K.
†Exp. indicates the power of 10 by which the value is multiplied.

Table 2

Wallops Island Soundings of 7 December 1963.

Height	Parameter*	Method A Grenade Data	Exp.†	Percent Difference From Pitot	Method B Grenade Data	Exp.†	Percent Difference From Pitot	Pitot-Static Data	Exp.†
36 km	pressure	.42947	3	2.01	.54489	3	29.43	.421	3
	density	.63440	-2	9.00	.80488	-2	38.30	.582	-2
	temp.	.23585	3	-6.48	.23585	3	-6.48	.2522	3
40 km	pressure	.24762	3	1.48	.31420	3	28.77	.244	3
	density	.34718	-2	-0.24	.44054	-2	26.59	.348	-2
	temp.	.24847	3	1.79	.24847	3	1.79	.2441	3
50 km	pressure	.64476	2	0.12	.81762	2	26.96	.644	2
	density	.89773	-3	2.13	.11384	-2	29.51	.879	-3
	temp.	.25022	3	-1.99	.25022	3	-1.99	.2553	3
60 km	pressure	.16392	2	-0.65	.20793	2	26.02	.165	2
	density	.23286	-3	-1.33	.29538	-3	25.16	.236	-3
	temp.	.24524	3	0.22	.24524	3	0.22	.2447	3
70 km	pressure	.38496	1	-3.76	.48829	1	22.07	.400	1
	density	.60302	-4	-3.05	.76428	-4	22.97	.622	-4
	temp.	.22240	3	-0.94	.22240	3	-0.94	.2245	3
77 km	pressure	.13965	1	-6.28	.17725	1	18.96	.149	1
	density	.21535	-4	-5.13	.27332	-4	20.41	.227	-4
	temp.	.22593	3	-1.68	.22593	3	-1.68	.2298	3

*Pressure in newtons/m^2, density in kg/m^3, temperature in °K.
†Exp. indicates the power of 10 by which the value is multiplied.

clearly that the use of an observed reference pressure is more accurate than assuming a standard atmosphere reference pressure when computing pressure and density profiles from a temperature profile. The differences between the pitot-static tube data and the profiles computed by Method A are less than half as large in every case as the differences between the pitot-static tube data and the profiles computed by Method B. These, as well as several more numerical calculations performed with actual soundings, simply confirm the fact expressed by Equation 8 that, in contrast to the sphere and pitot-static tube experiments, the error introduced in the grenade experiment by the choice of the initial value in the integration is not rendered negligible in the integration process.

There is, therefore, no advantage to assuming a pressure at the upper limit of the temperature profile in the grenade experiment. Rather, a pressure point measured by a balloon-sonde ascent should be used to compute the pressure profile upward. It can be stated with a high level of confidence that the error in the balloon-sonde pressure measurement is on the order of a few percent since the independent and simultaneous pitot-static tube observations in Tables 1 and 2 agree closely with the grenade data computed with Method A.

The small disagreement between grenade Method A and pitot-static data can be explained as experimental error. Neither technique is able to produce results with an error of less than several percent* (Reference 5). The data in Table 1, where the disagreement in results is generally larger than in Table 2, were taken with a developmental pitot-static tube experiment which may account for the larger discrepancies. The gauges used in the pitot-static experiment were improved after the Table 1 data were taken. In Table 2, the data from the two techniques agree quite closely, bearing out the improvement in the instrumentation.

An error is also introduced into the pressure and density profiles derived from grenade experiment temperatures because of the gap in the temperature profile that normally exists between the top balloon-sonde temperature observation and the lowest grenade temperature measurement. This error is difficult to assess accurately, but experimentation with numerical analyses using the standard atmosphere to bridge this gap has led to an estimate of less than two percent for such errors.

CONCLUSIONS

Several procedures are available for deriving pressure and density profiles from a measured temperature profile in the atmosphere. Basically, all of these procedures involve the vertical integration of the measured temperature profile to obtain the pressure profile once a reference pressure is known (i.e., assumed or measured). Variations of the hydrostatic equation and the perfect gas law are the means by which temperature and pressure profiles can be determined from a measured density profile. However, it is concluded that while integrating measured density downward from an assumed initial temperature soon converges to an accurate temperature (or pressure, as the case may be) profile as is done in the falling-sphere and pitot-static tube experiments, integrating a measured temperature profile yields the most accurate pressure and density results only when it is integrated upward from an initial measured pressure as in the grenade experiment. The magnitude of the total error introduced by the grenade experiment procedure in computing pressure is on the order of less than five percent as determined by independent observations and theoretical considerations.

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